Chapter 5
Gases - 2
Combined Gas Law, Ideal Gas Law and Applications of Gas Laws

Dr. Sapna Gupta

Combined Gas Law

The volume of a sample of gas at constant pressure is inversely proportional to the pressure and directly proportional to the absolute temperature.

The mathematical relationship: $V \propto \frac{T}{P}$

In equation form: $\frac{PV}{T} = \text{constant}$

$$\frac{P_i V_i}{T_i} = \frac{P_f V_f}{T_f}$$

Include the Avogadro's law and it becomes (*1=initial and 2= final*)

$$\boxed{\frac{P_i V_i}{n_i T_i} = \frac{P_f V_f}{n_f T_f}} \quad \text{OR} \quad \boxed{\frac{P_1 V_1}{n_1 T_1} = \frac{P_2 V_2}{n_2 T_2}}$$

Solved Problem:
Divers working from a North Sea drilling platform experience pressure of 5.0×10^1 atm at a depth of 5.0×10^2 m. If a balloon is inflated to a volume of 5.0 L (the volume of the lung) at that depth at a water temperature of 4°C, what would the volume of the balloon be on the surface (1.0 atm pressure) at a temperature of 11°C?

V_i = 5.0 L
P_i = 5.0×10^1 atm
T_i = 4° C = 277 K

V_f = ?
P_f = 1.0 atm
T_f = 11° C = 284 K

$$V_f = \frac{T_f P_i V_i}{T_i P_f}$$

$$V_f = \frac{(284\ \text{K})(5.0 \times 10^1\ \text{atm})(5.0\ \text{L})}{(277\ \text{K})(1.0\ \text{atm})}$$

$= 2.6 \times 10^2$ L
(2 significant figures)

Ideal Gas Law

Ideal Gas Law

- Combining the historic gas laws yields:

Boyle's law:	$V \propto \frac{1}{P}$
Charles's law:	$V \propto T$
Avogadro's law:	$V \propto n$

$$\rightarrow \quad V \propto \frac{nT}{P}$$

- Adding the proportionality constant, *R*

$$V = R\frac{nT}{P} \quad \rightarrow \quad PV = nRT$$

$$R = \frac{PV}{nT}$$

Molar Gas Constant in Various Units

Value of *R*
0.082058 L·atm/(K·mol)
8.3145 J/(K·mol)*
8.3145 kg·m^2/(s^2·K·mol)
8.3145 kPa·dm^3/(K·mol)
1.9872 cal/(K·mol)*

Standard Temperature and Pressure (STP)

- The reference condition for gases, chosen by convention to be exactly **0°C and 1 atm pressure**.
- The ideal gas equation is *not* exact, but for most gases it is quite accurate near STP (760 torr (1 atm) and 273 K)
- An “ideal gas” is one that “obeys” the ideal gas equation.
- At STP, **1 mol** of an ideal gas occupies **22.41 L**.

at STP

T = 0°C = 273 K

n = 1 mol

P = 1 atm

V = 22.41 L

Solved Problem:
A steel cylinder with a volume of 68.0 L contains O_2 at a pressure of 15,900 kPa at 23ºC. What is the volume of this gas at STP?

$$P_1 = 15{,}900\ \text{kPa} \times \frac{1\ \text{atm}}{101.3\ \text{kPa}} = 157.0\ \text{atm}$$

$T_1 = 23 + 273 = 296\ \text{K}$

$V_1 = 68.0\ \text{L}$

$P_2 = 1\ \text{atm}$

$T_2 = 273\ \text{K}$

$V_2 = ?$

$$\frac{P_1V_1}{n_1T_1} = \frac{P_2V_2}{n_2T_2}$$

$$V_2 = \frac{P_1V_1T_2}{T_1P_2} = \frac{(157.0\ \text{atm})(68.0\ \text{L})(273\ \text{K})}{(296\ \text{K})(1\ \text{atm})} = 9850\ \text{L}$$

Solved Problem:
A 50.0-L cylinder of nitrogen, N_2, has a pressure of 17.1 atm at 23°C. What is the mass of nitrogen in the cylinder?

V = 50.0 L
P = 17.1 atm
T = 23°C = 296 K

$$n = \frac{PV}{RT}$$

$$n = \frac{(17.1\ \text{atm})(50.0\ \text{L})}{\left(0.08206\ \frac{\text{L} \bullet \text{atm}}{\text{mol} \bullet \text{K}}\right)(296\ \text{K})} = 35.20\ \text{mols}$$

$$\text{mass} = 35.20\ \text{mol}\ \frac{28.02\ \text{g}}{\text{mol}}$$

mass = 986 g
(3 significant figures)

Solved Problem:
For an ideal gas, calculate the pressure of the gas if 0.215 mol occupies 338 mL at 32.0ºC.

n = 0.215 mol

V = 338 mL = 0.338 L

T = 32.0 + 273.15 = 305.15 K

P = ?

$$PV = nRT \Rightarrow \qquad P = \frac{nRT}{V}$$

$$P = \frac{(0.215\ \text{mol})\left(0.08206\ \frac{\text{L}\times\text{atm}}{\text{mol}\times\text{K}}\right)(305.15\ \text{K})}{0.338\ \text{L}}$$

= 15.928

= 15.9 atm

Gas Density and Molar Mass

- Using the ideal gas law, it is possible to calculate the moles in 1 L at a given temperature and pressure. The number of moles can then be converted to grams (per liter).

<u>Relation to density</u>

- Get n/V on one side (mol/vol)

$$\frac{n}{V} = \frac{P}{RT}$$

- Multiply by molar mass *M* (g/mol)

$$\boxed{\mathcal{M} \times \frac{n}{V}} = \frac{P}{RT} \times \mathcal{M}$$

- Units of density are g/L here.

$$d = \frac{P\mathcal{M}}{RT}$$

<u>To find molar mass</u>, rearrange the above equation.

$$\mathcal{M} = \frac{dRT}{P}$$

Solved Problem:
What is the density of methane gas (natural gas), CH_4, at 125°C and 3.50 atm?

M_m = 16.04 g/mol
P = 3.50 atm
T = 125° C = 398 K

$$d = \frac{M_m P}{RT}$$

$$d = \frac{(16.04\ \frac{\text{g}}{\text{mol}})(3.50\ \text{atm})}{\left(0.08206\ \frac{\text{L} \bullet \text{atm}}{\text{mol} \bullet \text{K}}\right)(398\ \text{K})}$$

$$d = 1.72\ \frac{\text{g}}{\text{L}}$$

(3 significant figures)

All Gas Laws

- Gases are compressible because the gas molecules are separated by large distances.
- The magnitude of P depends on how often and with what force the molecules strike the container walls.
- At constant T, as V increases, each particle strikes the walls less frequently and P decreases. (Boyle's Law)
- To maintain constant P, as V increases T must increase; fewer collisions require harder collisions. (Charles' Law)
- To maintain constant P and T, as V increases n must increase. (Avogadros' Law)
- Gas molecules do not attract or repel one another, so one gas is unaffected by the other and the total pressure is a simple sum. (Dalton's Law)

Key Points

- Combined gas law
- The ideal gas law
- Standard Temperature and Pressure
- Applications of gas laws
    - Density
    - Molar mass