Chapter 5 - Gases

Section 2 - Combined Gas Law, Ideal Gas Law and Applications

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Introduction

Once we know all the gas laws, we can combine them to see how all properties are related to each other. At present, we only know how volume is changed when pressure, temperature and mols are changed but what is the relationship between pressure and temperature? This is learned in combined gas law. We can study other relationships also e.g., with pressure and mols.

Combined gas law also helps us to study the changes in other properties of gases when two or more properties are changed at the same time, for example how the volume is affected when temperature and pressure are changed.

Once we combine all the laws to give one, then we can also learn to determine the volume or pressure etc. of a gas at any time, given the other data. This we will learn in ideal gas law.

Combined Gas Law

Boyle's Law
$PV = constant$

Charles Law
$\frac{V}{T} = constant$

Avogadro's Law
$\frac{V}{n} = constant$

Combining the three laws, volume is the common factor. The volume of a sample of gas at constant pressure is inversely proportional to the pressure and directly proportional to the absolute temperature and mols.

In equation form:

$$\frac{PV}{nT} = \text{constant}$$

$$\frac{P_i V_i}{n_i T_i} = \frac{P_f V_f}{n_i T_f}$$

Where,
"i" is initial and
"f" is final

Either of the two equations can be used as convenient.
(*1=initial and 2= final*)

$$\frac{P_i V_i}{n_i T_i} = \frac{P_f V_f}{n_f T_f}$$

$$\frac{P_1 V_1}{n_1 T_1} = \frac{P_2 V_2}{n_2 T_2}$$

Solved Problem: Combined Gas Law

Divers working from a North Sea drilling platform experience pressure of 50.0 atm at a depth of 500.0 m. If a balloon is inflated to a volume of 5.00 L (the volume of the lung) at that depth at a water temperature of 4.00 °C, what would the volume of the balloon be on the surface (1.00 atm pressure) at a temperature of 11.00°C?

V_i = 5.00 L
P_i = 50.0 atm
T_i = 4°C + 273 = 277 K

V_f = ?
P_f = 1.00 atm
T_f = 11°C + 273 = 284 K

$$\frac{P_iV_i}{n_iT_i} = \frac{P_fV_f}{n_fT_f}$$

$$V_f = \frac{P_iV_iT_f}{T_iP_f}$$

$$V_f = \frac{(284\ K)(50.0\ atm)(5.00\ L)}{(277\ K)(1.00\ atm)} = 2.60 \times 10^2\ L$$

Ideal Gas Law

Ideal Gas Law

In this law we remove the proportionality constant for the individual laws. Combining all the gas laws yields:

Boyle's law: $V \propto \frac{1}{P}$

Charles's law: $V \propto T$

Avogadro's law: $V \propto n$

$\longrightarrow V \propto \frac{nT}{P}$

To remove the proportionality symbol, a proportionality constant is added to equation. This constant is R, the gas constant.

$$V = R\frac{nT}{P} \longrightarrow PV = nRT$$

Gas Constant and Standard Temperature and Pressure (STP)

- To find the value of R, all other values must be known, For this we need to have standard numbers for all other measurements.
- These are called the standard temperature pressure conditions, which is exactly **0°C and 1 atm pressure**.
- The ideal gas equation is *not* exact, but for most gases it is quite accurate near STP (760 torr (1 atm) and 273 K)
- An "ideal gas" is one that "obeys" the ideal gas equation.
- At STP, **1 mol** of an ideal gas occupies **22.41 L**.
- Values of R depends on the units of all the quantities.

STP conditions
T = 0°C = 273 K
n = 1 mol
P = 1 atm
V = 22.41 L

$$R = \frac{PV}{nT}$$

Values of R
0.08206 L atm/(mol K)
8.3145 J/(mol K)
8.3145 kg $m^2/(s^2$ mol K)
1.987 cal/mol K

Solved Problem: Combined Gas Law using STP

A cylinder with a volume of 68.0 L contains O_2 at a pressure of 15,900 kPa at 23ºC. What is the volume of this gas at STP?

$$P_1 = 15{,}900\text{ kPa} \times \frac{1\text{ atm}}{101.3\text{ kPa}} = 157.0\ atm$$

$$T_1 = 23 + 273 = 296\text{ K}$$

$$V_1 = 68.0\text{ L}$$

$$P_2 = 1\text{ atm}$$

$$T_2 = 273\text{ K}$$

$$V_2 = ?$$

$$\frac{P_1V_1}{n_1T_1} = \frac{P_2V_2}{n_2T_2}$$

$$V_2 = \frac{P_1V_1T_2}{T_1P_2} = \frac{(157.0\ \cancel{\text{atm}})(68.0\ \mathbf{L})(273\ \cancel{\text{K}})}{(296\ \cancel{\text{K}})(1\ \cancel{\text{atm}})} = \boxed{9850\ L}$$

Solved Problem: Ideal Gas Law

For an ideal gas, calculate the pressure of the gas if 0.215 mol occupies 338 mL at 32.0ºC.

n = 0.215 mol

V = 338 mL x 10^{-3} = 0.338 L

T = 32.0 + 273.15 = 305.15 K

P = ?

$$PV = nRT$$

$$P = \frac{nRT}{V}$$

$$P = \frac{(0.215\ \text{mol})\left(0.08206\frac{\text{atm L}}{\text{mol K}}\right)(305.15\ \text{K})}{0.338\ \text{L}} = 15.928 = 15.9\ \text{atm}$$

Solved Problem: Ideal Gas Law

A 50.0-L container of nitrogen, N_2, has a pressure of 17.1 atm at 23°C. What is the mass of nitrogen in the cylinder?

V = 50.0 L

P = 17.1 atm

T = 23°C + 273 = 296 K

$$PV = nRT$$

$$n = \frac{PV}{RT}$$

$$n = \frac{(17.1\ \text{atm})(50.0\ \text{L})}{\left(0.08206 \frac{\text{atm L}}{\text{mol K}}\right)(296\ \text{K})} = 35.20 \text{ mols}$$

$$\text{mass} = 35.20\ \text{mol} \times \frac{28.02\ \text{g}}{1\ \text{mol}} = \boxed{986\ \text{g}}$$

Gas Density and Molar Mass

Using the ideal gas law, it is possible to calculate the moles in 1 L at a given temperature and pressure. The number of moles can then be converted to grams (per liter).

Relation to density

- Get n/V on one side (mol/vol)

$$\frac{n}{V} = \frac{P}{RT}$$

- Multiply by molar mass *M* (g/mol)

$$M \times \frac{n}{V} = \frac{P}{RT} \times M$$

- Units of density are g/L here.

$$d = \frac{PM}{RT}$$

To find molar mass, rearrange the above equation.

$$M = \frac{dRT}{P}$$

Solved Problem: Calculating gas density

What is the density of methane gas (natural gas), CH_4, at 125°C and 3.50 atm?

$M_m\ CH_4$ = 16.04 g/mol

P = 3.50 atm

T = 125 °C + 273 = 398 K

$$d = \frac{MP}{RT}$$

$$d = \frac{\left(16.04\frac{g}{mol}\right)(3.50\ atm)}{\left(0.08206\frac{atm\ L}{mol\ K}\right)(398\ K)} = 1.72\ g/L$$

Solved Problem: Calculating molar mass

Calculate the molecular mass of a liquid which when vaporized at 99.0 ^{o}C and 716 Torr gives 285 mL of vapor with a mass of 1.103 g.

Mass = 1.103 g

T = 99.0 ^{o}C + 273 = 372 K

V = 285 mL = 0.285 L

$$P = 716 \text{ torr} \times \frac{1 \text{ atm}}{760 \text{ torr}} = 0.942\ atm$$

$$M = \frac{dRT}{P}$$

$$\text{d} = \frac{m}{V}$$

$$\text{M} = \frac{\text{mRT}}{\text{VP}}$$

$$\text{M} = \frac{(1.103\ \text{g})\left(0.08206 \frac{\text{L}\bullet\text{atm}}{\text{mol}\bullet\text{K}}\right)(373\ \text{K})}{(0.285\ \text{L})\ (0.942\ \text{atm})} = 125.75 = \boxed{126\ \text{g/mol}}$$

Key Points

- Combined gas law
- The ideal gas law
- Standard Temperature and Pressure
- Applications of gas laws
  - Density
  - Molar mass