

Chapter 1

Solved Problems

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Significant Figures

How many significant figures are in each of the following measurements?

- a. 310.0 kg
- b. 0.224800 m
- c. 0.05930 kg
- d. 4.380×10^{-8} m
- e. 3.100 s
- f. 91,000

- a. 4 significant figures
- b. 6 significant figures
- c. 4 significant figures

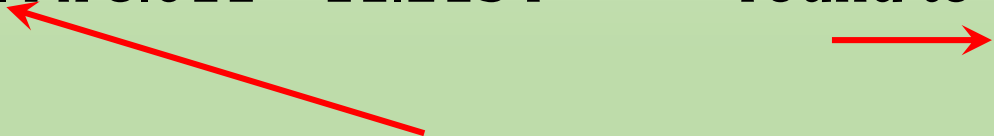
- d. 4 significant figures
- e. 4 significant figures
- f. 2 significant figures

Significant Figures

Addition and subtraction: Answer cannot have more digits to the right of the decimal than any of original numbers: e.g.

$$\begin{array}{r} 102.50 \text{ two digits after decimal point} \\ + \underline{0.231} \text{ three digits after decimal point} \\ \hline 102.731 \text{ round to } 102.73 \end{array}$$

Multiplication and division: Final answer contains the smallest number of significant figures: e.g.

$$1.4 \times 8.011 = 11.2154 \quad \text{round to } 11$$


(Limited by 1.4 to two significant figures in answer)

Significant Figures

Perform the calculations to the correct number of significant figures.

(a) $6.78 \times 5.903 \times (5.489 - 5.01)$

(b) $19.667 - (5.4 \times 0.916)$

Do the step in parentheses first. Use the subtraction rule to mark 0.479 to two decimal places since 5.01, the number in the parentheses with the least number of decimal places, has two.

Then perform the multiplication and round the answer to two significant figures since the number with the least number of significant figures has two.

Do the step in parentheses first. The number with the least number of significant figures within the parentheses (5.4) has two, so mark the answer to two significant figures.

Then perform the subtraction and round the answer to one decimal place since the number with the least number of decimal places has one.

Solution

$$\begin{aligned} \text{(a)} \quad & 6.78 \times 5.903 \times (5.489 - 5.01) \\ & = 6.78 \times 5.903 \times (0.479) \\ & = 6.78 \times 5.903 \times 0.479 \\ & 6.78 \times 5.903 \times 0.4790 = 19.1707 \\ & \qquad \qquad \qquad = 19 \end{aligned}$$

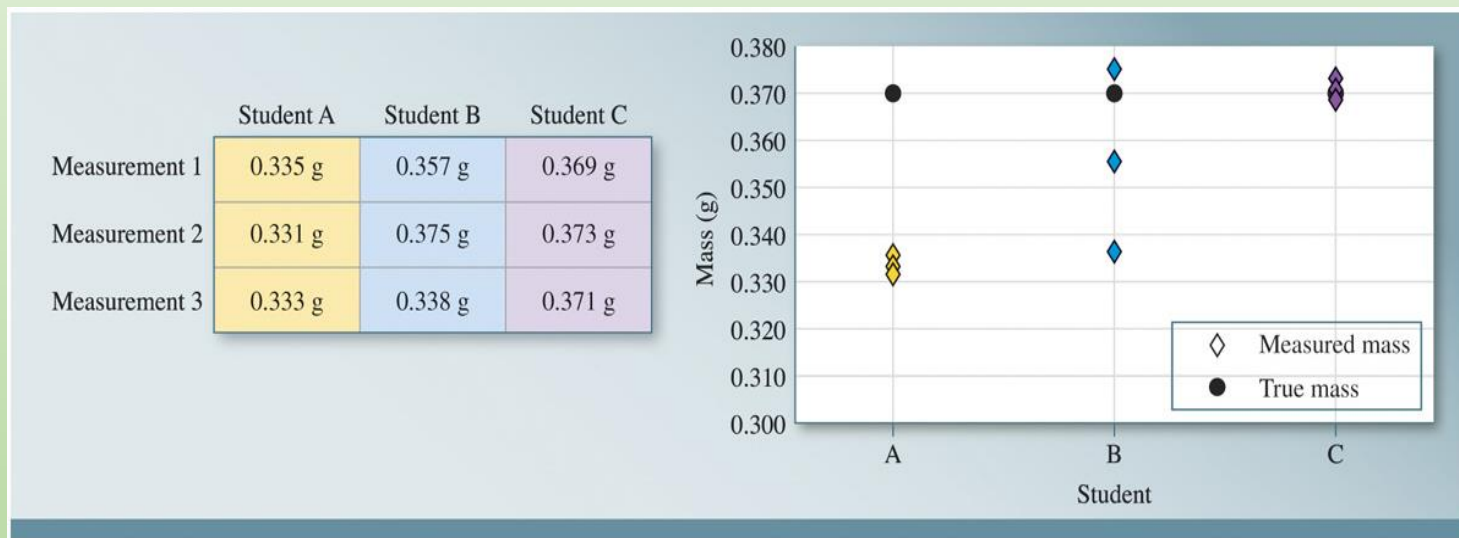
$$\begin{aligned} \text{(b)} \quad & 19.667 - (5.4 \times 0.916) \\ & = 19.667 - (4.9464) \\ & = 19.667 - 4.9464 \\ & 19.667 - 4.9464 = 14.7206 \\ & \qquad \qquad \qquad = 14.7 \end{aligned}$$

Accuracy and Precision

Describe accuracy and precision for each set

Student A	Student B	Student C
0.335 g	0.357 g	0.369 g
0.331 g	0.375 g	0.373 g
0.333 g	0.338 g	0.371 g
Average:		
0.333 g	0.357 g	0.371 g

- True mass is 0.370 grams



Student A's results are precise but not accurate.

Student B's results are neither precise nor accurate.

Student C's results are both precise and accurate.

Unit Conversion

A sample of sodium metal is burned in chlorine gas, producing 573 mg of sodium chloride. How many grams and kilograms is this?

$$1 \text{ mg} = 10^{-3} \text{ g}$$

and

$$1 \text{ kg} = 10^3 \text{ g}$$

$$573 \text{ mg} \times \frac{10^{-3} \text{ g}}{1 \text{ mg}}$$

$$573 \times 10^{-3} \text{ g}$$

$$0.573 \text{ g}$$

$$0.573 \text{ g} \times \frac{1 \text{ kg}}{10^3 \text{ g}}$$

$$0.573 \times 10^{-3} \text{ kg}$$

$$5.73 \times 10^{-4} \text{ kg}$$

Unit Conversion

An experiment calls for 54.3 mL of ethanol. What is this volume in cubic meters?

$$1 \text{ mL} = 1 \text{ cm}^3$$

$$(1 \text{ cm})^3 = (10^{-2} \text{ m})^3$$

$$1 \text{ mL} = 1 \text{ cm}^3 = 10^{-6} \text{ m}^3$$

$$54.3 \text{ mL} \times \frac{10^{-6} \text{ m}^3}{1 \text{ mL}}$$

$$54.3 \times 10^{-6} \text{ m}^3$$

$$5.43 \times 10^{-5} \text{ m}^3$$

Unit Conversion

The Star of Asia sapphire in the Smithsonian Institute weighs 330 carats (three significant figures). What is this weight in grams? One carat equals 200 mg (exact).

$$1 \text{ carat} = 200 \text{ mg (exact)}$$

$$1 \text{ mg} = 10^{-3} \text{ g}$$

$$330. \text{ carats} \times \frac{200 \text{ mg}}{1 \text{ carat}} \times \frac{10^{-3} \text{ g}}{1 \text{ mg}}$$

$$66000 \times 10^{-3}$$

$$6.60 \times 10^1 \text{ g} = 66.0 \text{ g}$$

Unit Conversion

The dimensions of Noah's ark were reported as 3.0×10^2 cubits by 5.0×10^1 cubits. Express this size in units of feet and meters. (1 cubit = 1.5 ft)

$$1 \text{ cubit} = 1.5 \text{ ft}$$

$$3 \text{ ft} = 1 \text{ yd}$$

$$1 \text{ yd} = 0.9144 \text{ m (exact)}$$

$$3.0 \times 10^2 \text{ cubits} \times \frac{1.5 \text{ ft}}{1 \text{ cubit}}$$

$$5.0 \times 10^1 \text{ cubits} \times \frac{1.5 \text{ ft}}{1 \text{ cubit}}$$

$$4.5000000 \times 10^2 \text{ ft}$$

$$7.5000000 \times 10^1 \text{ ft}$$

$$4.5 \times 10^2 \text{ ft}$$

by

$$7.5 \times 10^1 \text{ ft} = 75 \text{ ft}$$

$$4.5 \times 10^2 \text{ ft} \times \frac{1 \text{ yd}}{3 \text{ ft}} \times \frac{0.9144 \text{ m}}{1 \text{ yd}}$$

$$75 \text{ ft} \times \frac{1 \text{ yd}}{3 \text{ ft}} \times \frac{0.9144 \text{ m}}{1 \text{ yd}}$$

$$1.37160000 \times 10^2 \text{ m}$$

$$22.8600000 \text{ m}$$

$$1.4 \times 10^2 \text{ m}$$

by

$$23 \text{ m}$$

Density

The density of a piece of copper wire is 8.96 g/cm^3 . Calculate the volume in cm^3 of a piece of copper with a mass of 4.28 g .

$$d = \frac{m}{V}$$

$$V = \frac{m}{d} = \frac{4.28 \text{ g}}{8.96 \frac{\text{g}}{\text{cm}^3}} = 0.478 \text{ cm}^3$$

Density and Unit Conversions

A 23.5-kg sample of ethanol is needed for a large-scale reaction. What volume in liters of ethanol should be used? The density of ethanol is 0.789 g/cm^3 .

SORT

- Scan the problem for one or more numbers and their associated units. This number (or numbers) is (are) the starting point(s) of the calculation. Write them down as given.
- Scan the problem to determine what you are asked to find. Sometimes the units of this quantity are implied; other times they are specified. Write down the quantity and/or units you are asked to find.

STRATEGIZE

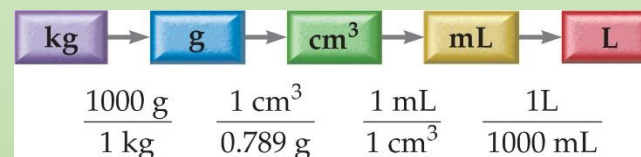
- For problems involving only conversions, focus on units. The solution map shows how to get from the units in the given quantity to the units in the quantity you are asked to find.
- For problems involving equations, focus on the equation. The solution map shows how the equation takes you from the given quantity (or quantities) to the quantity you are asked to find.
- Some problems may involve both unit conversions and equations, in which case the solution map employs both of the above points.

GIVEN: 23.5 kg ethanol

$$\text{density} = 0.789 \text{ g/cm}^3$$

FIND: volume in L

SOLUTION MAP



RELATIONSHIPS USED

$$0.789 \text{ g/cm}^3 \text{ (given in problem)}$$

$$1000\text{g} = 1 \text{ kg (Table 2.2)}$$

$$1000 \text{ mL} = 1 \text{ L (Table 2.2)}$$

$$1 \text{ mL} = 1 \text{ cm}^3 \text{ (Table 2.3)}$$

Continued

SOLVE

- For problems involving only conversions, begin with the given quantity and its units. Multiply by the appropriate conversion factor(s), canceling units, to arrive at the quantity you are asked to find.
- For problems involving equations, solve the equation to arrive at the quantity you are asked to find. (Use algebra to rearrange the equation so that the quantity you are asked to find is isolated on one side.) Gather each of the quantities that must go into the equation in the correct units. (Convert to the correct units using additional solution maps if necessary.) Finally, substitute the numerical values and their units into the equation and compute the answer.
- Round the answer to the correct number of significant figures.

SOLUTION

$$23.5 \cancel{\text{kg}} \times \frac{1000 \cancel{\text{g}}}{1 \cancel{\text{kg}}} \times \frac{1 \cancel{\text{cm}^3}}{0.789 \cancel{\text{g}}} \times \frac{1 \cancel{\text{ml}}}{1 \cancel{\text{cm}^3}} \times \frac{1 \text{L}}{1000 \cancel{\text{mL}}} = 29.7845 \text{ L}$$
$$29.7845 \text{ L} = 29.8 \text{ L}$$

Density and Unit Conversions

A 55.9-kg person displaces 57.2 L of water when submerged in a water tank. What is the density of the person in grams per cubic centimeter?

SORT

- Scan the problem for one or more numbers and their associated units. This number (or numbers) is (are) the starting point(s) of the calculation. Write them down as given.
- Write down the quantity and/or units you are asked to find.

GIVEN: $m = 55.9 \text{ kg}$

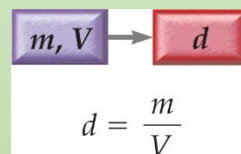
$V = 57.2 \text{ L}$

FIND: density in g/cm^3

STRATEGIZE

- For problems involving only conversions, focus on units. The solution map shows how to get from the units in the given quantity to the units in the quantity you are asked to find.
- For problems involving equations, focus on the equation. The solution map shows how the equation takes you from the given quantity (or quantities) to the quantity you are asked to find.
- Some problems may involve both unit conversions and equations, in which case the solution map employs both of the above points.

SOLUTION MAP



RELATIONSHIPS USED

$$d = \frac{m}{V} \text{ (definition of density)}$$

Continued


STRATEGIZE

- For problems involving only conversions, focus on units. The solution map shows how to get from the units in the given quantity to the units in the quantity you are asked to find.
- For problems involving equations, focus on the equation. The solution map shows how the equation takes you from the given quantity (or quantities) to the quantity you are asked to find.

SOLVE

- For problems involving only conversions, begin with the given quantity and its units. Multiply by the appropriate conversion factor(s), canceling units, to arrive at the quantity you are asked to find.
- For problems involving equations, solve the equation to arrive at the quantity you are asked to find. Gather each of the quantities that must go into the equation in the correct units. (Convert to the correct units using additional solution maps if necessary.) Finally, substitute the numerical values and their units into the equation and compute the answer.
- Round the answer to the correct number of significant figures.

SOLUTION MAP


$$d = \frac{m}{V}$$

RELATIONSHIPS USED

$$d = \frac{m}{V} \text{ (definition of density)}$$

The equation is already solved for the find quantity. Convert mass from kilograms to grams.

$$\begin{aligned} m &= 55.9 \text{ kg} \times \frac{1000 \text{ g}}{1 \text{ kg}} \\ &= 5.59 \times 10^4 \text{ g} \end{aligned}$$

Convert volume from liters to cubic centimeters.

$$\begin{aligned} V &= 57.2 \text{ L} \times \frac{1000 \text{ mL}}{1 \text{ L}} \times \frac{1 \text{ cm}^3}{1 \text{ mL}} \\ &= 57.2 \times 10^3 \text{ cm}^3 \end{aligned}$$

Compute density:

$$\begin{aligned} d &= \frac{m}{V} = \frac{55.9 \times 10^3}{57.2 \times 10^3 \text{ cm}^3} \\ &= 0.9772727 \frac{\text{g}}{\text{cm}^3} \\ &= 0.977 \frac{\text{g}}{\text{cm}^3} \end{aligned}$$

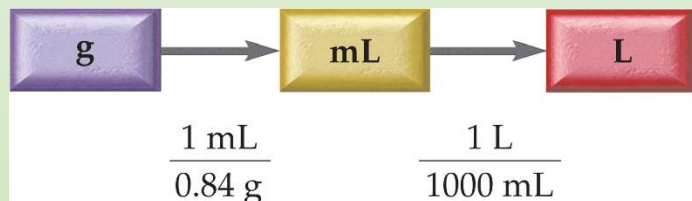
Density as Unit Conversion Factor

What is the volume in liters of 321 g of a liquid with a density of 0.84 g/mL?

GIVEN: 321 g

FIND: volume in L

SOLUTION MAP



RELATIONSHIPS USED

0.84 g/mL (given in the problem)

1L = 1000 mL

SOLUTION

$$321 \text{ g} \times \frac{1 \text{ mL}}{0.84 \text{ g}} \times \frac{1 \text{ L}}{1000 \text{ mL}} = 0.382 \text{ L} = 0.38 \text{ L}$$

The answer is in the correct units. The magnitude seems right because the density is slightly less than 1; therefore the volume (382 mL) should be slightly greater than the mass (321 g).